Palsule Equation for Tensile Modulus of Short Fibre Reinforced Polymer Composites and its Validation

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SUMMARY

Palsule equation for tensile Modulus of short fibre reinforced polymer composites has been derived as the first equation that accounts for the aspect ratio of reinforcer as follows:

$$E_c = \frac{1}{8}\left[E_m V_m + E_f V_f\right] + \frac{1}{4}\left[\frac{\left(E_f + \lambda E_m\right) + \lambda V_f\left(E_f - E_m\right)}{\left(E_f + \lambda E_m\right) - \psi V_f\left(E_f - E_m\right)}\right] E_m$$

Palsule equation has been validated by a comparison of the modulus of several short fiber reinforced polymer composites predicted by Palsule equation with their literature reported experimentally measured modulus values indicating that values predicted by Palsule equation are comparable with literature reported experimentally measured tensile modulus.

INTRODUCTION

Short fibre reinforced polymer composites are emerging as materials for commodity, engineering and high tech applications. In a short fibre reinforced polymer composite, a polymer matrix is reinforced by inorganic, organic, synthetic or natural short fibres. Short fibre reinforced polymer composites are high performance materials with light weight and high specific strength and stiffness [1]. In short fibre/polymer composites, matrix binds the fibres, prevents them from external factors, and transfers load to fibres. Fibres carry

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the load as the stress applied is carried by fibres, possibly equally by all fibres. Fiber/matrix interfacial adhesion determines the stress transfer to fibres and stress carried by fibres. Thus, the properties in general, and mechanical properties in particular, of short fibre reinforced polymer composites [2] are governed by properties of matrix, properties, amount and aspect ratio (length to diameter ratio) of reinforcer, and the interfacial adhesion between the fibre and the matrix. Modulus and strength of short fibre reinforced polymer composite increases with increasing fibre content, but beyond particular fibre content, only stiffness increases as there is no proportionate increase in strength. Tensile modulus is an important property of short fibre reinforced polymer composites.

Equations to Predict Tensile Modulus of Short Fibre Polymer Composites

There has been great interest to develop equations to predict the tensile modulus of short fibre reinforced polymer composites. The equations help predict the tensile modulus of a composite with known volume fractions of matrix polymer and reinforcing fibre, and also help predict the amount of reinforcing fibre required to develop a short fibre composite of a certain tensile modulus. Literature reports several models and equations to predict tensile modulus of short fibre polymer composites, and these include, Rule of mixture, Series model, Hirsch model [3], Halpin-Tsai equations [4-6], and the Nielson model [7-9]. None of these models and equations take into account the aspect ratio (length to diameter ratio) of the reinforcing fibre. Palsule equation is developed and derived herewith taking into consideration the aspect ratio (length to diameter ratio) of the reinforcing fibre. Palsule equation has been validated by comparing the modulus of several short fiber reinforced polymer composites predicted by Palsule equation with their literature reported experimentally measured modulus values.

Palsule Equation for Tensile Modulus of Short Fiber Reinforced Polymer Composites

Following is the Palsule equation for tensile modulus of short fibre polymer composites, and is derived below:

$$E_c = \frac{1}{8}\left[E_m V_m + E_f V_f\right] + \frac{1}{4}\left[\frac{\left(E_f + \lambda E_m\right) + \lambda V_f\left(E_f - E_m\right)}{\left(E_f + \lambda E_m\right) - \psi V_f\left(E_f - E_m\right)}\right]E_m \qquad (1)$$

Where E_c, E_m and E_f are the moduli of composite material, matrix and fiber respectively, V_m and V_f are the volume fraction of matrix and fiber respectively, λ is aspect ratio of the fibre and ψ is a constant = 0.5. The Palsule equation is unique as it takes into account the aspect ratio (l/d ratio) of the reinforcing fiber.

The derivation of Palsule equation is as follows. Lavengood and Goettler equation for tensile modulus of a three dimensionally oriented quasi-isotropic short fiber reinforced polymer composites:

$$E_c = K_L E_L + K_T E_T \tag{2}$$

Where, E_c is the modulus of the composite, K_L and K_T are constants, E_L is the longitudinal modulus and E_T is the transverse modulus of uniaxially oriented short fiber reinforced polymer composite. Following is the modified version of above Equation (2) for tensile modulus of a three dimensionally oriented quasi-isotropic short fiber reinforced polymer composites:

$$E_c = \frac{1}{8} E_L + \frac{1}{4} E_T \tag{3}$$

Where, E_c is the modulus of the composite, K_L and K_T are constants with values of 1/8 and 1/4 respectively; and E_L is the longitudinal modulus and E_T is the transverse modulus of uniaxially oriented short fiber reinforced polymer composite.

E_L, the longitudinal modulus can be calculated from the following rule of mixture:

$$E_L = E_m V_m + E_f V_f \tag{4}$$

E_T, the transverse modulus can be calculated from the modified Halpin-Tsai equations [4-6]:

$$E_T = E_M \left[\frac{1 + \varepsilon \lambda V_f}{1 - \varepsilon \psi V_f} \right] \tag{5}$$

Where, E_T is the transverse modulus of the composite, E_m is the modulus of the matrix, V_f and λ are the volume fractions and aspect ratio of the reinforcing fibre, ψ is a constant (=0.5) and ε is a ratio as indicated below:

$$\varepsilon = \left[\frac{E_f / E_m - 1}{E_f / E_m + \lambda} \right]$$

$$\text{or, } \varepsilon = \left[\frac{E_f - E_m}{E_f + \lambda E_m} \right] \qquad (6)$$

Substituting the value of ε in (5):

$$E_T = E_m \left[\frac{1 + \left(\frac{E_f - E_m}{E_f + \lambda E_m} \right) \lambda V_f}{1 - \left(\frac{E_f - E_m}{E_f + \lambda E_m} \right) \psi V_f} \right]$$

$$E_T = E_m \left[\frac{\left(E_f + \lambda E_m\right) + \lambda V_f \left(E_f - E_m\right)}{\left(E_f + \lambda E_m\right) - \psi V_f \left(E_f - E_m\right)} \right] \qquad (7)$$

Substituting the values K_L = 1/8, of K_T =1/4 and of E_L from equation (4) and of E_T from equation (7) in equation (3) Palsule equation is obtained as:

$$E_c = \frac{1}{8}\left[E_m V_m + E_f V_f\right] + \frac{1}{4}\left[\frac{\left(E_f + \lambda E_m\right) + \lambda V_f \left(E_f - E_m\right)}{\left(E_f + \lambda E_m\right) - \psi V_f \left(E_f - E_m\right)} \right] E_m \qquad (1)$$

Validation of the Palsule Equation for Tensile Modulus of Short Fiber Reinforced Polymer Composites

Palsule equation (1) above has been applied to predict the modulus of several short fiber reinforced polymer composites reported in literature, and the predicted value is compared with the experimentally measured value reported in literature. **Table 1** lists experimentally measured tensile modulus of various short fibre/polymer composites and tensile modulus values predicted by Palsule equation. **Table 1** indicates that tensile modulus of sisal fiber/low density polyethylene LDPE [10] composites predicted by Palsule equation (205 MPa) is comparable with the literature reported [10] experimentally measured tensile modulus (210MPa). **Table 1** also indicates that tensile modulus of various

Table 1. Experimentally measured tensile modulus of various short fiber/polymer composites and tensile modulus values predicted by Palsule equation

Composite Composition	Details and Volume Fraction of Fiber	Details and Volume Fraction of Matrix	Experimentally Measured Modulus	Modulus Predicted By Palsule Equation
Sisal Fiber/ Low Density Polyethylene LDPE [10]	**Sisal Fiber** Length = 6 mm Diameter = 200μm, Aspect Ratio λ=30 Modulus=15 GPa Volume Fraction 0.06	**Low Density Polyethylene LDPE** Density = 0.96 Modulus = 0.14GPa Volume Fraction = 0.94	210 MPa	205 MPa
Hemp Fiber/ High Density Polyethylene HDPE [11, 12]	Hemp Fiber Length =0.79 mm Diameter=0.678 mm Aspect Ratio λ = 12 Modulus=69 GPa Volume Fraction 0.06	High Density Polyethylene HDPE Modulus = 1.07 GPa Volume Fraction 0.94	~1 GPa	0.92 GPa
Rice Hull Fiber/ High Density Polyethylene HDPE [11, 12]	**Rice Hull Fiber** Diameter=0.55 mm Length =1.30 mm Aspect Ratio λ = 2.36 Modulus=22 GPa Volume Fraction 0.19	High Density Polyethylene HDPE Modulus = 1.07 GPa Volume Fraction 0.81	~1.6 GPa	1.033 GPa
Hemp Fiber/ High Density Polyethylene HDPE [11, 12]	**Hemp Fiber** Length =0.79 mm Diameter=0.678 mm Aspect Ratio λ = 12 Modulus=69 GPa Volume Fraction 0.19	High Density Polyethylene HDPE Modulus = 1.07 GPa Volume Fraction 0.81	~2.5 GPa	2.063 GPa

Table 1. Cont'd...

Composite Composition	Details and Volume Fraction of Fiber	Details and Volume Fraction of Matrix	Experimentally Measured Modulus	Modulus Predicted By Palsule Equation
Rice Hull Fiber/ High Density Polyethylene HDPE [11, 12]	**Rice Hull Fiber** Length =1.30 mm, Diameter=0.55 mm Aspect Ratio λ = 2.36 Modulus=22 GPa Volume Fraction 0.27	High Density Polyethylene HDPE Modulus = 1.07 GPa Volume Fraction 0.73	~1.8 GPa	1.3 GPa
Jute Fiber/ High Density Polyethylene HDPE [13]	**Jute Fiber** Length = 6mm Diameter = 50μm, Aspect Ratio λ =120 Modulus=26.5 GPa Volume Fraction 0.07	**High Density Polyethylene HDPE** Density = 0.96 Modulus = 9.4GPa Volume Fraction = 0.07	319.10 MPa	428 MPa
Banana Fiber/ Epoxy [14]	**Banana Fiber** Length =15 mm, Diameter=375 μm, Aspect Ratio λ = 40 Modulus=16000-20000MPa) Volume Fraction 0.06	**Epoxy** Modulus = 300MPa Volume Fraction = 0.94	410 MPa	364 MPa
Hemp Fiber/ High Density Polyethylene HDPE [11, 12]	**Hemp Fiber** Length =0.79 mm Diameter=0.678 mm Aspect Ratio λ = 12 Modulus=69 GPa Volume Fraction 0.27	High Density Polyethylene HDPE Modulus = 1.07 GPa Volume Fraction 0.73	~3 GPa	2.75 GPa

hemp fiber/polyethylene [11, 12] composites predicted by Palsule equation as 0.92 GPa, 2.063 GPa, and 2.75 GPa are comparable with the literature reported [11, 12] experimentally measured tensile modulus values of 1 GPa, 2.5 GPa, and 3 GPa respectively. Further, **Table 1** also indicates that tensile modulus of various rice hull fiber/polyethylene [11, 12] composites predicted by Palsule equation as 1.033 GPa and 1.3 GPa are comparable with the literature reported [11, 12] experimentally measured tensile modulus values of 1.6 GPa and 1.8 GPa respectively. Similarly, **Table 1** further indicates that tensile modulus of various jute fiber/high density polyethylene [13], and banana fiber/epoxy [14] composites predicted by Palsule equation as 428 MPa and 364 MPa are comparable with the literature reported [13, 14] experimentally measured tensile modulus values of 319.1 MPa and 410 MPa respectively. The above study and analysis of data in **Table 1** confirms that tensile modulus of several short fibre/polymer composites predicted by Palsule equation are comparable with their literature reported experimentally measured tensile modulus.

Several assumptions of Palsule equation applicable for ideal composites are not possible in processed and fabricated composites, like, uniform fibre distribution in composite, perfectly isotropic composite, perfect fibre/matrix interfacial adhesion, full load transfer from fibre to matrix, absence of voids in composites, etc. and practically, it is not possible to have all fibres with exactly same aspect ratio. Within the limits of their assumptions, Palsule equation makes a reasonable and an acceptable prediction of modulus values of short fibre reinforced polymer composites, at least for composites with low fibre volume fractions.

CONCLUSIONS

Palsule equation for tensile Modulus of short fibre reinforced polymer composites has been derived and validated as the first equation that accounts for the aspect ratio of reinforcer. Tensile modulus of several short fibre polymer composites predicted by Palsule equation are comparable with their literature reported experimentally measured tensile modulus.

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